Buenos Aires Semester in Computability, Complexity and Randomness

January to June 2013

Final Report

1 Introduction

The Buenos Aires Semester in Computability, Complexity and Randomness was an international gathering of Mathematicians and Computer Scientists that took place during the first half of 2013.

The focus was on topics in Computability Theory, Randomness, Computable Structures and Reverse Mathematics, with eyes open for conceptual and practical computational applications.

Activities consisted of a graduate-level course, a weekly seminar on topics covered in-depth over the period of the program, and a weekly seminar on individual research interests and related open problems. Most of the schedule was open for small-group research.

The Semester was held from January 8 to June 28, 2013. It counted in total 45 participants from 15 countries.

2 Organizers

International Organizers

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International Sponsors

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Local Sponsors

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Consejo Nacional de Investigaciones Científicas y Técnicas) (CONICET)

Universidad de Buenos Aires

3 Institutions

Argentina

Universidad de Buenos Aires and CONICET

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Harvard University
Indian River State College
Massachusetts Institute of Technology
University of California, Berkeley
University of Chicago
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5 Location

URL: http://www-2.dc.uba.ar/ccr/

The center of activity for the Semester was

Polo Científico Tecnológico (ex Bodegas Giol),
Godoy Cruz 2370, First and Second Floor
(C1425FQD) Ciudad Autónoma de Buenos Aires
Argentina

The course "Combinatorics, Complexity and Logic" was held in

Facultad de Ciencias Exactas y Naturales
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(C1428EGA) Ciudad Autónoma de Buenos Aires
Argentina

6 Statistics

The Semester was held from January 8 to June 28, 2013. It counted in total 45 participants from 15 countries, with the following distribution.

6.1 Participants per Nationality

Nationality	#Participants	%
Argentina	7	15.56%
Brazil	1	2.22%
Bulgaria	2	4.44%
Canada	2	4.44%
England	1	2.22%
France	2	4.44%
Germany	1	2.22%
Israel	2	4.44%
Italy	2	4.44%
Netherlands	1	2.22%
New Zealand	1	2.22%
Russia	2	4.44%
Singapore	1	2.22%
USA	19	42.22%
Uruguay	1	2.22%
Total	45	100.00%

6.2 Length of Stay

Period	#Participants	%
less than 1 month	18	40.00%
1 month–2 months	4	8.89%
3 months–4 months	5	11.11%
4 months–5 months	5	11.11%
5 months–6 months	13	28.89%
Total	45	100.00%

6.3 Age of Participants

Age	#Participants	%
20-29 years old	13	28.89%
30-39 years old	22	48.89%
40-49 years old	3	6.67%
50-59 years old	4	8.89%
60-69 years old	2	4.44%
more than 80 years old	1	2.22%
Total	45	100.00%

6.4 Gender of Participants

Gender	#Participants	%
female	8	17.78%
male	37	82.22%
Total	45	100.00%

7 Seminars and Talks

Seminar on Open Questions

Coordinated by Noam Greenberg and Antonio Montalbán.

Once a week, January to June 2013.

Current research was presented and discussed in this seminar. Stand-alone talks were given mostly by short-term visitors. Speakers presented the work that they were doing at the time and the most recent developments within it. The presentations provided an opportunity to discuss techniques and arguments in some detail. Active audience engagement was encouraged.

Seminar on Selected Topics

Coordinated by Joe Miller.

Twice a week, January to June 2013.

This seminar was a sequence of mini-courses in which the participants took turns presenting on a specific topic, with each topic spanning several meeting times. The emphasis was on mastering the material in depth. Topics included randomness extractors and expander graphs, randomness and computable analysis, and conservation results in reverse mathematics.

7.1 Higher randomness and triviality

Laurent Bienvenu – 2013-01-22

Abstract. Higher randomness theory investigates the notions of effective randomness one obtains when replacing "computable" by "hyperarithmetic" and "c.e." by Π^1_1 in the usual definitions (Martin-Löf randomness, Schnorr randomness, computable randomness, etc). After recalling the basics of the theory, I will present some recent work in collaboration with Noam Greenberg and Benoit Monin. The main question we will address is the following: do the (very rich) interactions between randomness and Turing degrees have a counterpart in the higher computability? We will argue that this is indeed the case, provided one correctly translates the notion of Turing reduction in the higher setting. We will thus introduce the notion of higher Turing reduction and show that a significant part of the classical theory translates accordingly. However, we will also see that the two landscapes (classical and "higher") differ dramatically on some key aspects, such as the existence of a uniform oracle tests and measures. If time permits, I will discuss the impact this has on the study of lowness and triviality, and will ask some open questions.

7.2 Oberwolfach randomness and computing K-trivial sets

Noam Greenberg – 2013-01-23, 2013-01-24, and 2013-01-29

7.3 Partial orders and reverse mathematics

Alberto Marcone – 2013-01-30

Abstract. I'll present joint work with Emanuele Frittaion about the reverse mathematics of theorems about partial orders included in Frassé's book 'Theory of Relations' and originally due to Milner and Pouzet, Bonnet, and Erdös and Tarski. Some theorems deal with the existence of linear extensions that preserve some finiteness property, while the others are concerned with the structure and cardinality of the collection of initial intervals in partial orders without infinite antichains. We obtain that some statements are equivalent to ACA_0, others to ATR_0, and others to Σ^0_2-bounding. We have also statements for which we do not know the precise answer yet: they are provable in WKL_0 but not in RCA_0.

7.4 Lebesgue density in Π^0_1 classes (and K-triviality)

Joe Miller – 2013-02-01, and 2013-02-05

Abstract. The Lebesgue density theorem states that for almost every X in a measurable class C, the relative measure of C along X converges to one. Call such an X a density point of C. We ask how random must a real X be to be a density point of every Π^0_1 class that contains it. Along the way, we prove that no K-trivial can be cupped to 0' by an incomplete ML-random (one direction of the solution to the ML-cupping problem) and together with the work of Bienvenu, Greenberg, Kucera, Nies and Turetsky that Noam has presented, we prove that there is an incomplete ML-random that computes every K-trivial (solving the ML-covering problem). Although this talk is a closely related to Noam's lectures, no understanding of previous material should be required. (Joint work variously with Bienvenu, Hölzl and Nies; Day; Andrews, Cai, Diamondstone and Lempp.)

7.5 On Omega Degree Spectra

Alexandra Soskova – 2013-02-06

Abstract. We relativize the notion of degree spectra by considering multi-component spectra, i.e. a degree spectrum with respect to a given sequence of sets of natural numbers. We study this under the omega-enumeration reducibility. It is a uniform reducibility between sequences of sets of natural numbers, introduced and studied by Soskov, H. Ganchev, M. Soskova, etc. The notion of omega-degree spectrum generalizes the notion of relative spectrum. The omega-co-spectrum is the set of omega-enumeration degrees which are lower bounds of the elements of the omega-spectrum. We prove that most of the properties of the degree spectrum such as the minimal pair theorem and the existence of quasi-minimal degree are true for the omega-degree spectrum. We give an explicit form of the elements of the omega-co-spectrum of a structure by means of computable Σ^+_k formulae.

7.6 SJT as an analog of K-triviality

Daniel Turetsky – 2013-02-13

Abstract. SJT is a natural ideal strictly contained within the ideal of K-trivial degrees. We have been hearing about a number of results connecting the K-trivials with notions of randomness. For example, the covering problem which states that every K-trivial is computable from a difference random. Many of these results have analogs for SJT; for example, every SJT is computable from a Demuth random. I will present SJT and its parallels with K-triviality.

7.7 Mutual Information and Weak Lowness Notions

Ian Herbert – 2013-02-14

Abstract. We examine a definition of mutual information for reals due to Levin. This definition induces an associated lowness notion of having finite self-information, that is, mutual information with oneself. Hirschfeldt and Weber proved that the set of reals with finite self-information strictly contains the K-trivials. We show that it is in fact

much larger, constructing a perfect Π^0_1 class of such reals. The proof technique involves a certain more general class of weakenings of lowness for K, and we discuss some results regarding these notions and some other applications.

7.8 Computable Algebra: A Personal Perspective

Chris Conidis – 2013-03-05

Abstract. We will survey some recent results in computable algebra and give some possible new directions for research.

7.9 Generic computability, and the strange results that arise in its study

Greg Igusa – 2013-03-07

Abstract. We define a real to be generically computable if there is a partial recursive function which correctly computes almost all of the bits of the real without necessarily halting on all inputs, but also without giving incorrect answers on any inputs. Our definition is originally motivated by an empirically observed phenomenon in complexity theory: that certain problems are much easier to solve in practice than would be suggested by their complexity classification, however, we study generic computability from a purely recursion theoretic point of view, primarily to investigate and understand the strange properties that this notion has. Relativized generic computation is not transitive, and thus not a true reduction procedure. There are no minimal pairs (or even finite minimal sets) for generic computation. We briefly sketch some of the new obstacles and new tools that are present in this context. If we change our definition of relativized generic computation to make it transitive, then the equivalence classes are each uncountable, and the relationship becomes Π^1_1-complete. The Turing degrees can be shown to embed in the generic degrees, and under certain hypotheses, the generic degrees can be classified in terms of their relationships with the embedded Turing degrees.

7.10 A model-theoretic approach to characterizing randomness notions

Cameron Freer – 2013-03-13

Abstract. Given an unordered countable structure, when is a presentation of it algorithmically random? Computable invariant measures concentrated on the isomorphism class of the structure lead us to one possible answer to this question. Conversely, given such an invariant measure, we may ask which measure-one set of points in the sample space is mapped to the isomorphism class, giving rise to a notion of randomness via an "almost everywhere" theorem. Even for familiar examples of invariant measures, these randomness notions are not always easy to identify; on the other hand, we show how Martin-Löf randomness (and Kurtz randomness, upon broadening "isomorphism class" to "models of a theory") can be recovered in this way. We will describe several open questions about the notions of randomness characterizable in this setting. Joint work with Nate Ackerman.

7.11 Extensions of Levy-Shoenfield Absoluteness

Gerald Sacks – 2013-03-14

7.12 Four Lectures on Normal Numbers

Part I. Equivalent Definitions of Normality and Selected Theorems
Verónica Becher – 2013-03-19

Part II. Normality and Incompressibility by Finite Automata
Pablo Heiber – 2013-03-21

Part III. Absolute Normality and Normality in Different Bases
Verónica Becher and Ted Slaman – 2013-03-26

Part IV. Normality and Polynomial Time Martingales
Santiago Figueira (joint work with André Nies) – 2013-03-27

7.13 Differentiability and porosity

André Nies – 2013-04-03

Abstract. Brattka, Miller and Nies proved in 2011 that a real z is computably random iff every nondecreasing function is differentiable at z. I will consider the analogous theorem when the effectiveness condition on the function is varied.

1. The analogous theorem holds for polynomial time randomness and polynomial time computable nondecreasing functions.
2. For interval-c.e. function (essentially, the variation function of a computable function), the right randomness strength is Martin-Loef random reals at which the Lebesgue density theorem holds for effectively closed sets.

Surprisingly, the analytic notion of porosity plays a major role in both proofs.

7.14 A Computability Theoretic equivalent to Vaught's Conjecture

Antonio Montalbán – 2013-04-09, 2013-04-11, and 2013-04-16

Abstract. We find two computability theoretic properties on the models of a theory T which hold if and only if T is a counterexample to Vaught's conjecture.

7.15 Primes in Computable UFDs

Joe Mileti – 2013-04-10

Abstract. In algebraic number theory, one works in extensions of Z by algebraic integers and studies how the prime from Z factor (possibly into ideals when unique factorization fails) in this larger ring. We will show how to how to extend Z in such a way that we can control the primes in any Π_2 way, all while maintaining unique factorization. As a corollary, we establish the existence of a computable UFD such that the set of primes is Π_2-complete in every computable presentation.

7.16 Complex isomorphisms of simple computable structures

Alexander Melnikov – 2013-04-17

Abstract. A computable structure C is Δ^0_2-categorical if for every computable structure B isomorphic to C there exists a 0'-computable isomorphism f: B –> C. There

are also several uniform versions of this notion such as effective, uniform, and relative Δ^0_2-categoricity (to be defined). In this talk we address the following problem: Given a class K of computable structures, characterize all members of K which are (effectively/uniformly/relatively) Δ^0_2-categorical. The problem has already been studied for well-orderings (Ash), linear orders under some effective restrictions (McCoy), Boolean algebras (Harris), algebraic fields (Miller), and completely decomposable groups (Downey and M.). We restrict ourselves to (algebraically) simple classes, namely to computable equivalence relations and multi-cyclic abelian groups (those are direct sums of cyclic and quasi-cyclic groups). Cenzer, Calvert, Harizanov, and Morozov were the first to investigate categoricity notions in these classes. We will see that, rather unexpectedly, the study of Δ^0_2-categorical members in these classes requires new ideas and advanced recursion-theoretic techniques such as the 0"' priority method. Our machinery enables us to obtain several structural results and also answer questions which were left open in (Cenzer et al). Joint work with Rod Downey and Keng Meng (Selwyn) Ng.

7.17 The c.e. sets disjoint from an c.e. set A

Peter Cholak – 2013-04-18

Abstract. We will discuss an invariant on a c.e. set A determined by the sets disjoint from A. We explore this invariant's behavior on a nice definable collection of c.e. sets. There is a also a rich interplay between this invariant and Friedberg splits of c.e. sets.

7.18 Strong reductions between combinatorial problems

Damir Dzhafarov – 2013-04-23

Abstract. I will discuss recent investigations of various reducibility notions between Π^1_2 principles of second-order arithmetic, the most familiar of which is implication over the subsystem RCA_0. In many cases, such an implication is actually due to a considerably stronger reduction holding, such as a uniform (or Weihrauch) reduction. (Here, we say a principle P is uniformly reducible to a principle Q if there are fixed reduction procedures Phi and Gamma such that for every instance A of P, Phi(A) is an instance of Q, and for every solution S to Phi(A), Gamma(A + S) is a solution to A.) As an example, nearly all the implications between principles lying below Ramsey's theorem for pairs are uniform reductions. In general, the study of when such stronger implications hold and when they do not gives a finer way of calibrating the relative strength of mathematical propositions. In addition, this analysis sheds light on several open questions from reverse mathematics, including that of whether the stable form of Ramsey's theorem for pairs (SRT^2_2) implies the cohesive principle (COH) in ω (standard) models of RCA_0.

7.19 Random Measures

Quinn Culver – 2013-04-24

Abstract. We define a natural, computable map that associates to each real a Borel probability measure, so that we can talk about random measures, the images of the (ML) random reals. We show that such random measures are atomless and mutually singular with respect to Lebesgue. We introduce a certain property that lies strictly between atomlessness and absolute continuity (with respect to the Lebesgue measure) that we

conjecture random measures satisfy. We then discuss other maps, ask the question "Why is the first map more natural?", and discuss some ways in which that question might be precisifiable.

7.20 A Lightface Analysis of the Differentiability Rank

Linda Brown Westrick – 2013-04-25

Abstract. We examine the computable part of the differentiability hierarchy defined by Kechris and Woodin. In that hierarchy, the rank of a differentiable function is an ordinal less than omega_1 which measures how complex it is to verify differentiability for that function. We show that for each recursive ordinal alpha>0, the set of Turing indices of C[0,1] functions that are differentiable with rank at most alpha is Pi_2 alpha + 1-complete. This result is expressed in the notation of Ash and Knight.

7.21 Density-one points of Π^0_1 classes

Mushfeq Khan – 2013-04-30

Abstract. From previous talks this semester we have a good idea of how Martin-Löf random density-one points behave. For example, they form a proper subclass of the difference random reals (the so-called "Madison randoms"), and are hence incomplete. We investigate their behavior in the absence of randomness. We will show that the notions of dyadic density-one and full density-one are not the same (even though they coincide on the randoms). We will also show that every real is computable from a full density-one point.

7.22 C.e. equivalence relations under computable approximations

Luca San Mauro – 2013-05-15

Abstract. (joint work with Uri Andrews, Joe Miller, Keng Meng Ng, Steffen Lemmp, and Andrea Sorbi) The theory of c.e. equivalence relations (ceers) has been approached in different ways (for instance: looking for a computable analogue of Borel reducibility, or, as in Ershov's seminal work, considering the theory of numberings). In this talk, we focus on the degree structure P generated by the following reducibility: given two ceers R and S, we say that R is reducible to S if there exists a computable function f s.t. for every x,y, xRy iff f(x)Sf(y). Our presentation is divided in two parts. Firstly, through an examination of a particular class of ceers, we give a couple of general results on P: we show that P is neither an upper semilattice nor a lower semilattice, and that its first-order theory is undecidable. Then, we move our attention to universal ceers. We begin with an overview of two classical properties that imply universality (precompletness and e-completness) and we show the failure of Myhill Isomorphism Theorem in this context. Finally, we discuss the interplay between effective inseparability and universality for ceers, showing that every uniformly e.i. ceer is universal.

7.23 Martin-Lof Random Brownian Motion

Kelty Allen – 2013-05-16

Abstract. We define Martin-Lof random Brownian motion and investigate some of its

properties. We will cover some of the "almost surely" results from classical probability theory that hold for MLR Brownian motion, and cover some of the many interesting results about the zero set of a sample path. In particular, we show that the first zero after any computable real is layerwise computable, and discuss an application to the Dirichlet problem.

7.24 Lowness in recursive model theory

Johanna Franklin – 2013-05-21

Abstract. Lowness has been studied in the contexts of degree theory, learning theory, and randomness. I will discuss lowness in the context of recursive model theory: we say that a degree is low for isomorphism if, whenever it can compute an isomorphism between two recursively presented structures, there is actually a recursive isomorphism between them. I will describe the class of Turing degrees that are low for isomorphism, identify some particular subclasses, and show how it behaves with respect to measure and category.

7.25 Randomizable Reals

Ian Haken – 2013-05-22

Abstract. In 2008 Jan Reimann and Ted Slaman showed that every non-recursive real has a measure such that the real is not an atom of the measure and such that the real is Martin-Löf random relative to the measure. In this talk I will explore the question of whether or not this result can be raised to notions of randomness stronger than Martin-Löf, and discuss, relative to stronger notions of randomness, the problem of characterizing the set of reals for which such measures exist.

7.26 Coarse Computability and Algorithmic Randomness

Denis Hirschfeldt – 2013-05-28

Abstract. I will describe some interactions between algorithmic randomness and coarse computability, which make use of old and recent results on the interactions between random sets, highly nonrandom sets, and Turing reducibility. This is joint work with Carl Jockusch and Paul Schupp.

7.27 Relative computability of models of strongly minimal theories

Uri Andrews – 2013-05-29

Abstract. We will examine how complicated a model of a strongly minimal theory may be, given that the theory has at least one recursive model. In particular, we show that if a strongly minimal theory has a recursive model, then all models are recursively presentable in $0^{(4)}$. It is open whether or not 4 is sharp. (Work in progress, so caveat emptor. Joint with Julia F. Knight.)

7.28 The complexity of equivalence relations

Keng Meng Ng – 2013-05-30

Abstract. The study of the complexity of equivalence relations has recently been (somewhat) active, with computable, c.e. and other equivalence relations higher up the arithmetical hierarchy being the objects of study. The main reducibility amongst equivalence relations is given by the usual m-reducibility. This gives a natural way of comparing equivalence relations. We survey some recent results in this topic, including completeness results, results on other possible reducibilities and applications.

7.29 The Muchnik Lattice and Intuitionistic Logic

Rutger Kuyper – 2013-06-06

Abstract. The Medvedev and Muchnik lattices are an attempt to capture the computational content of proofs in intuitionistic logic using a 'calculus of problems'. While the lattices themselves turn out to fall short, Skvortsova remarkably showed that there exist factors of the Medvedev lattice which do capture intuitionistic propositional logic (IPC), while Sorbi and Terwijn later showed the analogous result for the Muchnik lattice. Unfortunately these factors are constructed in an ad hoc manner and do not have a clear computational motivation. In this talk I will present natural factors of the Muchnik lattice which capture IPC, using well-known concepts such as lowness, 1-genericity, hyperimmune-freeness and computable traceability.

7.30 Families of sets and their degree spectra

Iskander Kalimullin – 2013-06-10

Abstract. The talk will be devoted to the recent results on possible collection of degrees in which a fixed family can be uniformly enumerated.

8 Course on Combinatorics, Complexity and Logic

By Theodore Slaman.

Once a week, March to June 2013.

Course Description

The course explores the interplay between the specification of mathematical problems, especially in combinatorics, and the descriptive complexity of their solutions. Applied in one direction, we obtain simple combinatorial problems that can have an intrinsically complicated solutions, such as finding a Hamiltonian path in a graph or finding a homogeneous set for a computable partition. In the other direction, we combinatorially analyze semantics to exhibit limits on what can be described or what can be proven, such as in Razborov's theorems on the limits of definability by monotone circuits or as in the Paris-Harrington example of a natural incompleteness in elementary arithmetic.

Course Outline

1. Basic Ramsey Theory
2. Finite combinatorics and complexity theory
   (a) Introduction to circuit complexity
   (b) Razborov's Theorem
3. Infinitary combinatorics and computabilty theory
   (a) Introduction to computability theory
   (b) Computational analysis of Ramsey's Theorem
   (c) Ramsey's Theorem for Pairs in detail
4. Infinitary combinatorics with finitary consequences
   (a) Mathematical logic and formal theories of arithmetic
   (b) Incompleteness theorems
   (c) Conservation theorems for infinitary principles
   (d) The Paris-Harrington Theorem, a finite combinatorial principle not provable by elementary methods

Supplementary Reading Material

W. T. Gowers. Razborov's method of approximations. lecture notes, 2009.

David Seetapun and Theodore A. Slaman. On the strength of Ramsey's theorem. *Notre Dame J. Formal Logic*, 36(4):570-582, 1995.

Leo A. Harrington. A mathematical incompleteness in Peano arithmetic. In Jon Barwise, editor, *Handbook of mathematical logic*, volume 90 of *Studies in Logic and the Foundations of Mathematics*, pages 1133-1142. North-Holland Publishing Co., Amsterdam, 1977.

General References

Robert I. Soare. *Recursively enumerable sets and degrees.* Perspectives in Mathematical Logic. Springer-Verlag, Berlin, 1987.

H.-D. Ebbinghaus, J. Flum, and W. Thomas. *Mathematical logic.* Undergraduate Texts in Mathematics. Springer-Verlag, New York, second edition, 1994.

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9 Other Activities

The Buenos Aires Semester in Computability, Complexity and Randomness organized, as a special activity, the presentation of the film Codebreaker, the drama documentary about life and work of Alan Turing.

"Codebreaker" was directed by Clare Beavan and produced by Story Center Productions, Furnace y Channel 4, in 2011. Film duration 81 minutes.

Official site for the film
`http://www.turingfilm.com`

Site for the film at the Facultad de Ciencias Exactas y Naturales, UBA
`http://noticias.exactas.uba.ar/codebreaker-en-exactas`

The film was exhibited on April 10, 2013 at 18hs at Aula Magna del Pabellón 2, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Buenos Aires.

Buenos Aires, July 2013